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A general formula for non-cohesive bed load sediment transport

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Abstract

A bed load transport formula for non-cohesive sediment, based on the bed-shear concept of Meyer-Peter and Müller, was developed and validated for steady flows, oscillatory flows, and combined steady and oscillatory flows. The bed load formula introduced in this study was examined using data from experimental and field measurements for a wide range of flows and sediment conditions, as occurring in river, coastal, and marine environments. More than 1000 steady and 500 oscillatory flow cases were used in the study. The relationship between the bed load transport and the total Shields parameter to the power 1.5 was first confirmed for the steady flows. An exponential factor to take into account the effect of the critical Shields parameter was introduced. The proposed formula for the steady current was expanded and generalized to take into account the effects of oscillatory flows as well as oscillatory flows with a superimposed current at an arbitrary angle. The time-dependent bed load transport was treated in a "quasi-steady" manner using the quadratic value of the instantaneous Shields parameter for the two half-periods of the wave (when the total instantaneous velocity u is in the direction of the wave, $u > 0$, or in the opposite direction, $u < 0$). A good correlation was found between the bed load formula and the measurements for colinear oscillatory and steady flows when no phase-lag occurred in the experiments. However, a marked scatter was observed since the Shields parameter had to be estimated and not derived directly from measured data. Finally, the validity and limitations of the obtained bed load transport formula are discussed.

Keywords: sediment transport; bed load; non-cohesive sediment; current; waves; river; nearshore; laboratory data; field data

1. Introduction

Accurate prediction of sediment transport rates is an important element in morphological studies of river, coastal, and marine environments. Sediment transport occurs in two main modes: bed load and suspended load. The bed load is the part of the total load which is travelling immediately above the bed and is supported by intergranular collisions rather than fluid turbulence (Wilson, 1966). The suspended load, on the other hand, is the part of the load which is primarily supported by the fluid turbulence (Fredsøe and Deigaard, 1994). Thus, bed load includes mainly sediment transport for coarse materials (saltation) or fine material on plane beds (saltation at low shear stresses and sheet flow at high shear stresses), although both types of transport can occur together and the limit is not always easy to define. The earliest formulas (models) proposed to estimate bed load transport were mainly based on the concept that the sediment transport rate can be related to the bottom shear stress (Meyer-Peter and Müller, 1948; Einstein, 1950) and these formulas were valid for steady, uni-directional flows. In coastal and marine environments, the process of sediment transport becomes increasingly complex due to the presence of oscillatory flows, and the interaction between steady and oscillatory flows. For example, in longshore sediment transport, the influence of the short waves is expressed through a wave-induced sediment stirring that increases

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the bed-shear stress (Bijker, 1967; Watanabe, 1982; Van Rijn, 1993).

However, in the case of cross-shore sediment transport, a wave-averaged approach is not adequate due to the dominant role of the time-dependent oscillatory orbital motion near the sea bed, induced by the short waves. For example, the residual cross-shore transport of sand due to short-wave asymmetry is generally described using an intra-wave model concept, resolving the unsteady transport process through the wave cycle. Bagnold (1966) developed an "energetics transport model", in which the instantaneous transport through the wave cycle is related to the instantaneous energy dissipation rate due to bottom friction. Bagnold's approach formed the basis for several bed load transport models (Bailard and Inman, 1981; Ribberink, 1998) as well as total load transport models (Madsen and Grant, 1976; Bailard, 1981). These models use the "bed-shear stress concept" for river flow and the time-dependent transport through the wave cycle is treated in a quasi-steady way, i.e., the time-history effects from previous phases of the wave cycle or from previous wave cycles are neglected.

The development of practical sediment transport models still has a strong empirical character and relies heavily on physical insights and quantitative data as obtained in laboratory and field studies. The objective of this study is to develop a reliable, robust and general formula for predicting bed load transport for a wide range of river, coastal, and marine conditions. A large number of data sets are used for the model development including steady and unsteady flows.

Using the 'bed-shear stress concept', a major problem is to estimate the total shear stress at the bottom. As a first step in this study, only conduit, flume and river data for steady flows were studied to implement the bed load formula. For these kinds of experimental studies, the bed-shear stress may be estimated from energy slope measurements. Then, the formula was generalized to oscillatory flows and combined steady and oscillatory flows. However, for these more complex flows the bed-shear stress cannot be estimated directly from the measurements, but the stress must be calculated from theoretical models of the bed roughness. Thus, comparisons with experimental data were made using the skin friction as proposed by Soulsby (1997) and a semi-empirical function to estimate the bottom roughness height in the sheet flow regime from Wilson (1989).

2. Bed-load transport by currents

Here, bed load refers mainly to the rolling, sliding, and jumping grains in almost continuous contact with the bed, but also, following the ideas of Wilson (1966), to the upper-regime transport where a layer with a thickness of several grain diameters is transported and where intergranular collision is important (sheet flow).

2.1. Existing formulas

The bed load transport is often represented by the following non-dimensional parameter,

$$\Phi = \frac{q_{sb}}{\sqrt{(s-1)gd_{50}^3}} \tag{1}$$

where q_{sb} is the volumetric bed load sediment transport rate per unit time and width, s is the ratio between densities of sand and water, g is the acceleration due to gravity, and d_{50} is the median diameter. Some authors have also proposed to use the parameter $\Phi_b = q_{sb}/(W_s d_{50})$, where W_s is the settling velocity. If $d_{50} > 10^{-3}$ mm, $\Phi_b \approx \Phi$ as the settling velocity is proportional to the square-root of the median diameter (according to Van Rijn, 1984), whereas for $d_{50} < 10^{-3}$ mm, Φ_b becomes smaller than Φ implying that Φ_b introduces a characterization of the sediment transport rate more sensitive for fine sediments.

Several relationships for bed load transport under a steady current have been proposed where the rate is related to the dimensionless bottom shear stress or Shields parameter,

$$\theta_c = \frac{\tau_c}{(\rho_s-\rho)gd_{50}} = \frac{\frac{1}{2}f_c U_c^2}{(s-1)gd_{50}} \tag{2}$$

where τ_c is the shear stress at the bottom due to the current, ρ_s and ρ are the sediment and water density, respectively, f_c is the dimensionless friction factor, and U_c the current velocity. Three commonly applied formulas were investigated,

Meyer-Peter and Müller (1948) $\Phi = 8(\theta_c - \theta_{cr})^{3/2}$ (3)

Nielsen (1992) $\Phi = 12\theta_c^{0.5}(\theta_c - \theta_{cr})$ (4)

Ribberink (1998) $\Phi = 11(\theta_c - \theta_{cr})^{1.65}$ (5)

where θ_{cr} is the critical Shields parameter. The formula for θ_{cr} proposed by Soulsby and Whitehouse (1997, see also Soulsby, 1997, pp. 104–110) was used in this paper.

2.2. Comparison with experimental data

To investigate bed load transport formulas, a wide range of existing data sets were compiled and analyzed.

Table 1
Data summary for bed load sediment transport in a steady current

Author(s)	Flow type	Nbr	Material	s	d_{50} (mm)
Gilbert (1914)	Steady uniform flow, flume, plane bed	250	Sand	2.65	0.3–4.9
U.S. Waterways Experiment Station (1935–1936)	Steady uniform flow, flume, plane bed	162	Sand	2.65	0.18–4.1
Willis et al. (1972)	Flume experiment, plane bed	43	Sand	2.65	0.1
Brownlie (1981)	Various experimental data, plane bed	297	Plastic	1.30–1.41	2.2–20.2
			Sand	2.49–2.67	0.088–20
Brownlie (1981)	Various field data, plane bed	40	Sand	2.65	0.084–7.0
Smart (1984, 1997 with Nikora, 1999)	Exp. for steep channels and field data, plane bed	140	Sand	2.65	2.0–10.5
			Gravel	2.65	53–200
Nnadi and Wilson (1992)	Pressured closed conduit	105	Bakelite	1.55	0.67–1.05
			Sand	2.67	0.70
			Nylon	1.14	3.94

Table 1 summarizes these data sets, where the type of flow motion and sediment properties are listed. It may be noted that many data come from the compilation made by Brownlie (1981), where little indication was given about the type of sediment transport (bed load or/and suspended load), although comments on the presence of bedforms were provided. From these data, only plane-bed cases were selected where bed load should prevail. Typically, for fine sediment, suspended load is not negligible when bedforms occur.

Since uncertainties already exist in the measurements of the bed load sediment transport (especially for field measurements), a prediction within a factor 2 of the experimental data is usually considered to be a satisfactory result. In Table 2, the percentage of predicted values included within a factor of 2 or a factor of 5 deviation is presented (Px2 and Px5, respectively), as well as the root-mean-square error defined as $E_{\mathrm{rms}} = 1/n\sum(\log(q_{\mathrm{s,pred}}/q_{\mathrm{s,meas}}))^2$. To avoid any distortion in the calculation of E_{rms} for very low transport (where $q_{\mathrm{s,pred}}$ or $q_{\mathrm{s,meas}}$ could be found equal to zero), a minimum value for $q_{\mathrm{s,pred}}$ and $q_{\mathrm{s,meas}}$ is used ($q_{\mathrm{s,min}} = 10^{-8}\mathrm{m}^2/\mathrm{s}$). It appears that the Meyer-Peter and Müller formula, which was calibrated with coarse sediment data, gives fairly good results for most of the data, even for large shear stresses and fine sediment. An underestimation is, however, observed for the Willis et al. data, but as sediment used for this experiment was very fine (0.1 mm), some suspended transport probably occurred. The three formulas studied show equivalent behavior for large shear stresses with a slightly better prediction skill for the Ribberink formula. For smaller Shields parameter (θ close to θ_{cr}, where θ_{cr} is estimated using the Soulsby and Whitehouse formula), all formulas tend to overestimate the bed load rate with errors up to one order of magnitude for the Nielsen formula, which significantly increases E_{rms}-value. Thus, it seems that the use of the critical Shields parameter as a limit for no transport is not accurate enough in the existing formulas.

Table 2
Prediction of bed load transport rate within a factor of 2 and 5 of the measured values and root-mean-square errors using current data

Author(s)	All data			All data except Willis et al.		
	Px2 (%)	Px5 (%)	E_{rms}	Px2 (%)	Px5 (%)	E_{rms}
Meyer-Peter and Müller	62	83	0.34	66	87	0.30
Nielsen	54	72	0.48	57	75	0.46
Ribberink	66	85	0.28	69	89	0.25
Eq. (6)	73	89	0.19	78	93	0.15

2.3. A new formula for bed load transport

To better understand the effect of the critical value of the Shields parameter on the sediment transport, Φ-values were plotted versus the ratio $\theta_c/\theta_{\mathrm{cr}}$ in Fig. 1. The studied formulas are also plotted on the same graph using a constant value for $\theta_{\mathrm{cr}} = 0.04$ (mean value for the data set).

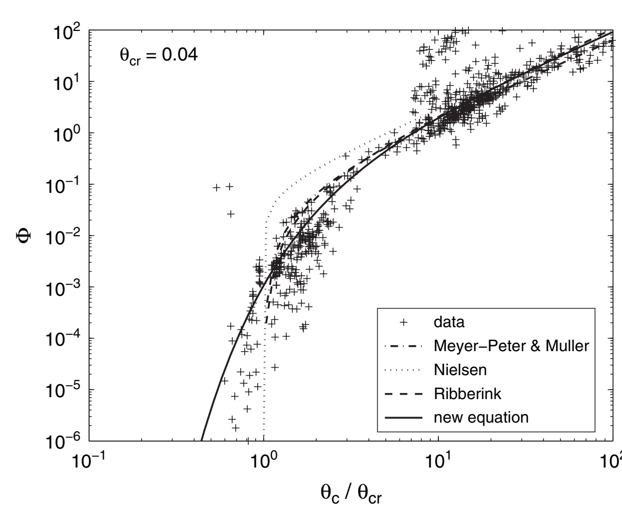

Fig. 1. Effect of the critical Shields parameter on bed load transport rate: comparison between data and the studied formulas.

All formulas are in a fairly good agreement with the data if $\theta_c > 5\theta_{cr}$, but they tend to slightly overestimate Φ when $\theta_{cr} < \theta_c < 5\theta_{cr}$. For this range of values, the Meyer-Peter and Müller and Ribberink formulas exhibit a better behavior than the Nielsen formula. However, when $\theta_c \lesssim \theta_{cr}$, significant deviations in the predictions occur. All these formulas predict no sediment transport, although low sediment transport is often observed. The prediction of the critical Shields parameter is obviously associated with marked uncertainties (so is the sediment transport estimation for these low values). Thus, to avoid such errors, a new approach was introduced using an exponential relationship for the effect of the critical Shields parameter, which allows low sediment transport when $\theta \approx \theta_{cr}$. A calibration with the data leads to the following bed load formula:

$$\Phi = 12\theta_c^{1.5} \exp\left(-4.5\frac{\theta_{cr}}{\theta_c}\right) \tag{6}$$

Cheng (2002) proposed a formula close to Eq. (6), but without taking into account the effect of the critical Shields parameter. Actually, the Cheng formula seems to include a constant value on $\theta_{cr} \approx 0.05$. The effect of the critical value of the Shields parameter is, however, not negligible as observed by Meyer-Peter and Müller (1948).

Figs. 1 and 2 indicate that this new approach regarding the effect of the critical Shields parameter significantly improve the prediction skill for low shear stresses. Predictions within a factor 2 deviation reach 70% (even 80% without Willis et al. data), which increase the results by 10% compared to previous formulas (cf. Table 2). The new relationship tends to overestimate the transport rate when $\theta_c \lesssim \theta_{cr}$ (see Fig. 1), but it corresponds to very low rates.

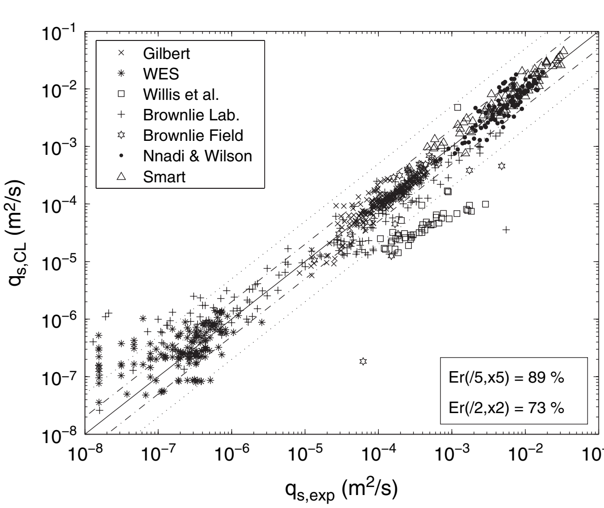

Fig. 2. Comparison between bed load transport predicted by the new formula (Eq. (6)) and experimental data.

3. Bed-load transport by waves

3.1. Existing formulas

Several relationships for bed load transport by waves have been proposed related to the wave orbital velocity at the bottom U_w or the wave Shields parameter θ_w defined as,

$$\theta_w = \frac{\frac{1}{2} f_w U_w^2}{(s-1)gd_{50}} \tag{7}$$

where f_w is the dimensionless wave friction factor. Assuming that the rough turbulent regime is fully developed, the friction factor can be estimated by the formula suggested by Swart (1974).

Formulas are often employed to estimate the wave half-cycle sediment transport $\Phi_{1/2}$ (Madsen and Grant, 1976; Soulsby, 1997; Soulsby et al., 1993). The net transport can then be calculated as the difference between the half-cycle transport beneath the crest and beneath the trough (the shear stress is calculated using the maximum and minimum values of the wave velocity at the bottom, $u_{w,max}$ and $u_{w,min}$, respectively, instead of the wave orbital velocity U_w). In some other studies (Bailard and Inman, 1981; Ribberink, 1998), an instantaneous relationship is introduced for the bed load $\Phi(t)$ that can be integrated over a wave period. The following formulas are among the most common ones for calculating sediment transport under waves:

Madsen and Grant (1976)

$$\Phi_{1/2} = 12.5\frac{W_s}{\sqrt{(s-1)gd_{50}}}\theta_w^3 \tag{8}$$

Bailard and Inman (1981)

$$\Phi(t) = \frac{\epsilon_b f_w}{(s-1)^2 g^2 \tan\phi d_{50}} u_w(t)^3 \tag{9}$$

Dibajnia and Watanabe (1992)

$$\Phi = 0.001\frac{W_s}{\sqrt{(s-1)gd_{50}}}\Gamma^{0.55}\frac{\Gamma}{|\Gamma|} \tag{10}$$

Soulsby (1997)

$$\Phi_{1/2} = 5.1(\theta_w - \theta_{cr})^{3/2} \tag{11}$$

Ribberink (1998)

$$\Phi(t) = 11(|\theta_w(t)| - \theta_{cr})^{1.65}\frac{\theta_w(t)}{|\theta_w(t)|} \tag{12}$$

It can be noted that Madsen and Grant and Dibajnia and Watanabe used the parameter Φ_b instead of the parameter Φ (cf. Section 2.1), which explains the coefficient $W_s/\sqrt{(s-1)gd_{50}}$ in their formulas. Bailard and Inman proposed a coefficient value of $\epsilon_b = 0.13$ and ϕ is the internal friction angle of the sediment ($\phi \approx 30°$). Dibajnia and Watanabe defined Γ as a function of the half-periods T_{wc} and T_{wt} and the amount of sediment entrained that settled in each half-period, since some of it might still be in suspension from the previous half-period (see also Camenen and Larroudé, 2003).

3.2. Development of a new formula

As Madsen (1991) or Ribberink (1998) proposed, the instantaneous sediment transport rate may be related to the instantaneous shear stress in the same manner as for the steady case. Following the idea of Dibajnia and Watanabe (1992), a simplified velocity profile at the bottom may be used to estimate the effect of the wave asymmetry on the sediment transport. Thus, the net sediment transport over a wave period is estimated for each half-period using a characteristic value on the quadratic velocity, or equivalently on the shear stress (if the friction coefficient is assumed to be constant). Thus, the mean value of the instantaneous shear stress over half a period may be used (see Fig. 3),

$$\begin{aligned}\theta_{w,onshore} &= \frac{1}{T_{wc}}\int_0^{T_{wc}} \theta_w(t)\mathrm{d}t \\ \theta_{w,offshore} &= \frac{1}{T_{wt}}\int_{T_{wc}}^{T_w} \theta_w(t)\mathrm{d}t\end{aligned} \tag{13}$$

where T_{wc} and T_{wt} are the half-periods where the instantaneous velocity $u_w(t)$ (or instantaneous Shields parameter) is onshore ($u_w(t) > 0$) or offshore ($u_w(t) < 0$), respectively, and the instantaneous shear stress is defined as follows:

$$\theta_w(t) = \frac{\frac{1}{2}f_w|u_w(t)|u_w(t)}{(s-1)gd_{50}} \tag{14}$$

Fig. 3 presents a typical velocity profile and the associated instantaneous Shields parameter profile over a wave period. In the case of an asymmetric wave, a maximum in the shear stress occurs during onshore flow (in the direction of the waves) that is larger than the minimum during offshore flow. This causes a net sediment transport in the direction of the waves.

A constant value on the friction coefficient over the wave period was assumed when calculating the Shields parameter (Eq. (13)). Drake and Calantoni (2001) and Antunes Do Carmo et al. (2003) showed that in general the wave friction coefficient depends also on the acceleration of the fluid near the bottom. This dependence was previously noted by Trowbridge and Madsen (1984a, b), who presented an analytical study of the turbulent wave boundary layer showing the importance of a time-varying eddy viscosity. However, it was assumed here that the predictions of the shear stresses using a constant friction factor are accurate enough for the sediment transport calculations.

Using the same approach as for the steady current, an equation for the net sediment transport under waves may be derived similar to Eq. (6). The net sediment transport under waves is expressed as,

$$\Phi = a\sqrt{\theta_{cw,onshore} + \theta_{cw,offshore}}\,\theta_{w,m}\exp\left(-b\frac{\theta_{cr}}{\theta_w}\right) \tag{15}$$

where a and b are coefficients ($|\theta_{cw,onshore}|$ is supposed to be always larger than $|\theta_{cw,offshore}|$) and $\theta_{w,m} = \langle|\theta_w(t)|\rangle$ is the time-averaged absolute value of the instantaneous Shields parameter.

A conceptual model that supports this type of formulation would be based on the following assumptions:

- the transport in the bottom layer is the product between the typical speed of the layer and the layer thickness, where the former is denoted U_s and the latter Δ_s;
- U_s is assumed proportional to the net shear velocity over a wave period, which gives the $|\theta_{w,onshore} + \theta_{w,offshore}|^{1/2}$-dependence ($u_* \propto \theta^{1/2}$);
- Δ_s is assumed proportional to the mean wave shear stress, which gives the $\theta_{w,m}$-dependence.

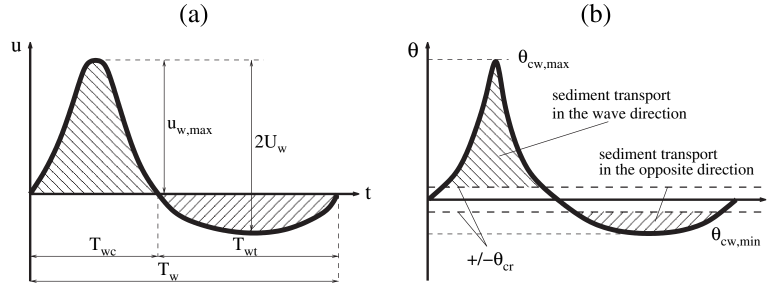

Fig. 3. (a) Typical wave velocity profile and (b) instantaneous Shields parameter profile, over a wave period in the direction of the waves.

The overall effect of the critical shear stress on the sediment transport over a wave period is estimated using the same approximation as for the steady current. However, since Soulsby (1997, pp. 104–106) proposed to compute the critical shear stress based on the maximum shear stress, the maximum Shields parameter is used in Eq. (15).

A difficulty when applying the formula (and any other formulas based on the shear stress) is to estimate the total Shields parameter (or total friction coefficient) for the sheet flow regime. The work by Wilson (1989) on wave-induced sheet flow roughness was employed in this study, where he proposed to use the same equation as for steady current but with the maximum wave-induced Shields parameter ($k_s = k_{s,W} = 5\theta_w d_{50}$). As most of the results presented by Ribberink (1998) were based on the skin friction, in order to be consistent, two computations were made, using first the skin friction ($k_s = 2d_{50}$) and then the Wilson equation. It should be noted that the Wilson formula requires an iterative approach when solving for k_s.

3.3. Comparison with experimental data

To investigate bed load transport under waves only, a wide range of existing data sets were compiled and analyzed. Table 3 summarizes these data sets, where the type of experiment, sediment properties, and wave properties are listed. It can be observed that most of the data are from oscillating water tunnels (OWT). This kind of experiment has two advantages for this study: large orbital velocities can be reached and bed load transport is prevailing. Previously, experimental studies were often carried out using an oscillating tray (OT; oscillating bed in a tank of still water, cf. Manohar, 1955; Kalkanis, 1964; Abou-Seida, 1965; Sleath, 1978).

Fig. 4 shows the calculated and measured bed load transport for the studied experimental cases using values on the empirical coefficients of $a = 6$ and $b = 4.5$. This new fit shows that if the effect of the critical Shields parameter on the mean sediment transport rate over a wave period is similar to steady flow, the total net rate is a function of the Shields parameter to the power 1.5 with a smaller value on the coefficient a than for steady current. Soulsby (1997) found similar results using the Meyer-Peter and Müller equation (Eqs. (3) and (11) include the coefficient values 8 and 5.1, respectively). This lower value may correspond to a phase-lag between instantaneous sediment concentration and the velocity at the bottom.

Around 60% of the cases are predicted within a factor of 2 of the measured values. As discussed previously, an additional problem when comparing the transport formula with the wave measurements is that no shear stress may be derived from the experimental measurements, but the shear stress has to be calculated based on an estimate of the bed roughness. This introduces an extra element of uncertainty in the calculations of the sediment transport rates. The formula proposed by Wilson (1989) was employed to calculate the roughness height, and an underestimation for the Ahilan and Sleath data set and an overestimation for the Sawamoto and Yamashita data set are observed. One explanation could be that large uncertainties are induced by this kind

Table 3
Data summary for bed load sediment transport in an oscillatory flow with and without current

Author(s)	Exp. Facil.	Cycle	Nbr	s	d_{50} (mm)	U_c (m/s)	$U_{w,max}$ (m/s)	T_w (s)
Kalkanis (1964)	OT	Half	25	2.63	1.68–2.82	0	0.28–0.71	3.2–6.2
Abou-Seida (1965)	OT	Half	34	2.65	0.14–2.61	0	0.35–1.28	1.7–5.1
			9	2.23	0.70	0	0.41–0.80	2.0–4.8
Sleath (1978)	OT	Half	22	2.60	1.89, 4.24	0	0.20–0.68	0.5–2.7
			12	1.14	3.04	0	0.07–0.17	1.3–9.0
Horikawa et al. (1982)	OWT	Half	6	2.66	0.2	0	0.76–1.27	2.6–6.0
Sawamoto and Yamashita (1986)	OWT	Half	22	1.58	1.5	0	0.74–1.25	3.8
				2.65	0.2–1.8	0	0.46–1.25	3.8
Ahilan and Sleath (1987)	OWT	Half	5	1.14	4.0	0	0.3–0.5	3.6–3.7
			4	1.44	4.3	0	1.1–1.2	4.7–4.9
Watanabe and Isobe (1990)	OWT	Full	11	2.65	0.18, 0.87	0	0.27–0.43	3.0, 6.0
			65	2.65	0.18, 0.87	−0.3–0.25	0.27–0.52	3, 6
King (1991)	OWT	Half	178	2.65	0.1–1.1	0	0.3–1.2	2.0–12.0
Dibajnia and Watanabe (1992)	OWT	Full	25	2.65	0.20	0	0.6–1.0	1.0–4.0
			76	2.65	0.20	−0.26–0.22	0.61–1.24	1–4
Ribberink and Chen (1993)	OWT	Full	4	2.65	0.128	0	0.6–1.2	6.5
Ribberink and Al Salem (1994)	OWT	Full	10	2.65	0.21	0	0.7–1.4	5.0–12.0
Delft Hydraulics (1993–1999)	OWT	Full	52	2.65	0.13–0.24	−0.45–0.56	0.37–1.49	5, 12
Dohmen-Janssen (1999)	OWT	Full	27	2.65	0.13–0.32	0.23–0.45	0.46–1.85	4–12
Dohmen-Janssen and Hanes (2002)	LWF	Full	4	2.65	0.21	−0.05	0.88–1.05	6.5, 9.1
Ahmed and Sato (2003)	OWT	Full	15	2.65	0.21–0.74	0	1.16–1.85	3.0

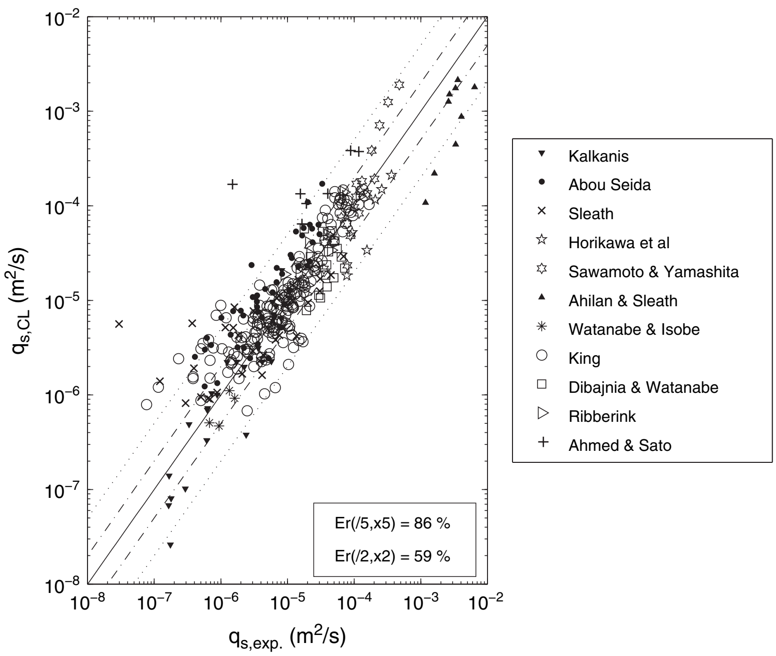

Fig. 4. Comparison between bed load transport predicted by the new formula (Eq. (15)) and experimental data with waves (k_s calculated using the Wilson formula, 1989).

of iterative formula (cf. Bayram et al., 2003). Using the skin roughness height shows better agreement for the Sawamoto and Yamashita data set, but worse agreement for the Ahilan and Sleath data set.

Finally, it may be noted that Eq. (15) slightly underestimates the sediment transport rate for the Kalkanis data set and overestimates the rate for the Abou-Seida and Sleath data sets. Thus, it appears that the OT set-up does not yield the same behavior concerning the sediment transport as the OWT set-up.

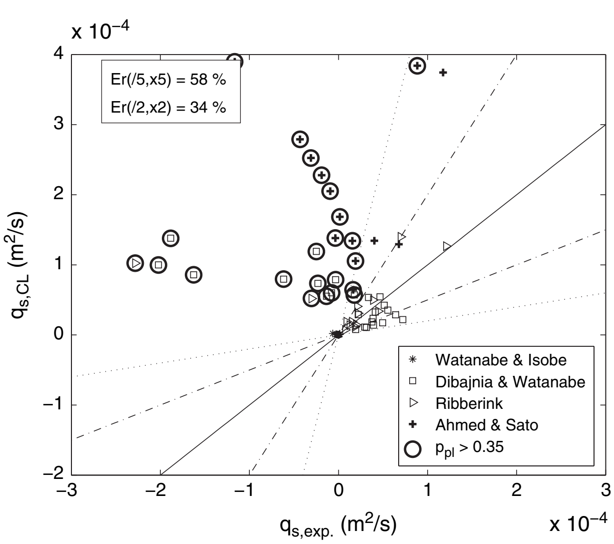

Fig. 5. Comparison between bed load transport predicted by the new formula (Eq. (15)) and experimental data over a full wave cycle (k_s calculated using the Wilson formula, 1989, $k_s = k_{s,W}$).

Another remark to be made (see Fig. 5) is that the direction of the net sediment transport may be opposite to the direction of the waves. Ribberink and Chen (1993) and Dibajnia and Watanabe (1992) observed this phenomenon for highly asymmetric waves and fine sediment ($d_{50} \leq 0.2$ mm). It corresponds to the phase-lag in the response of the sand to the fluid. The quantity of sediment in suspension depends not only on the instantaneous velocity, but also on the settling velocity. In the case of oscillating flows, not all the sand grains put into suspension during the first half-period settle during the same half-period. The proportion still in suspension is then carried away in the opposite direction during the second half-period. Dohmen-Janssen (1999) and Dohmen-Janssen et al. (2002) introduced a phase-lag parameter to describe the phase-lag effects,

$$p_{pl} = \frac{2\pi\delta_s}{W_s T_w} \tag{16}$$

where δ_s is the thickness of the sheet flow layer. Following Asano (1992) and Dohmen-Janssen et al. (2002), $\delta_s = 10 d_{50}\theta_w$. They observed that phase-lag effects occur when $p_{pl} > 0.3$–0.4. In Fig. 5 are the cases, where strong phase-lag occurs, emphasized (circles). These data points correspond to an overestimation (and/or wrong direction) of the predicted values using Eq. (15). Thus, it appears that the proposed formula is restricted to cases without phase-lag effects. This phenomenon is not yet included in the present formula but could be approximated by adding a correction factor (Dohmen-Janssen,

Table 4
Prediction of bed load transport rate within a factor of 2 and 5 of the measured values and root-mean-square errors using waves data

Author(s)	All data ($k_s = k_{s,W}$)			Full cycle data ($k_s = k_{s,W}$)			All data ($k_s = 2d_{50}$)		
	Px2 (%)	Px5 (%)	E_{rms}	Px2 (%)	Px5 (%)	E_{rms}	Px2 (%)	Px5 (%)	E_{rms}
Madsen and Grant	37	64	1.25	06	14	5.22	45	75	0.99
Bailard and Inman	47	75	0.92	38	57	3.81	45	73	0.93
Dibajnia and Watanabe	36	75	0.49	49	85	0.79	36	75	0.49
Soulsby (coefficient 2.5)	36	75	0.98	20	52	3.92	39	76	0.97
Ribberink	29	73	1.08	20	46	4.14	36	76	1.03
Eq. (15)	59	86	0.77	34	58	3.84	57	86	0.78

1999; Dohmen-Janssen et al., 2002; Camenen and Larroudé, 2003).

In Table 4, the percentage of predicted values within a factor of 2 deviation or a factor of 5 deviation and the root-mean-square error are presented for the six studied formulas and all the data, as well as for the data encompassing complete wave cycles, using the Wilson formula (1989) to compute the roughness height ($k_s = k_{s,W}$). The same calculations were also performed for all the data using the skin friction ($k_s = 2d_{50}$) to compute the Shields parameter.

Eq. (15) yields the best overall results, although for the full cycle data, since no phase-lag is taken into account, the results are poorer and the scatter is larger ($E_{rms} = 3.8$). The Madsen and Grant formula tends to overestimate the sediment transport for large values on the shear stress. This is because they employed a formula proportional to the Shields parameter to the power 3 and calibrated with lower shear stress data. Soulsby suggested to use the maximum shear stress in his formula, which causes a large overestimation of the transport rates. Using a coefficient value of 2.5 instead of 5.1 improves the results (cf. results in Table 4), but it still implies an overestimation of the sediment transport for low shear stresses. If the mean shear stress is used instead of the total shear stress (with the coefficient 5.1), results are better and quite similar to those using Eq. (15), except for data with low sediment transport rates, which are largely overestimated. The Bailard and Inman, Dibajnia and Watanabe, and Ribberink formulas also present overestimations for low sediment transport rates. This means that the effect of the critical Shields parameter is not taken into account properly (not included in the Bailard and Inman and Dibajnia and Watanabe formulas). The Ribberink formula, as it stands, generally overestimates the sediment transport rate. Using the coefficients (7.9, 1.97) instead of (11, 1.65) or using $k_s = d_{50}$, as Ribberink (1998) proposed, improves the results. However, this is not in accordance with the formula proposed for steady current (cf. Eq. (5)) or with the physical values of the roughness height. The Dibajnia and Watanabe formula, since it takes into account the effect of the phase-lag, allows the sediment transport to be in the opposite direction to the waves for fine sediment. Thus, it presents the overall best results for the full cycle data and a much lower scatter ($E_{rms} < 1$). On the other hand, it yields poor results for the half-cycle data. Finally, it should be noted that results from the OT data are generally more scattered. This may be a result of the shear stresses being close to the critical shear stress, inducing additional randomness, as well as uncertainties in the measurements.

The main theoretical difference between the Ribberink relationship (Eq. (12)) and Eq. (15) is that the former one is a mean value of the instantaneous sediment transport over the wave period, whereas the latter calculates the sediment transport from the mean shear stress. Using the mean value over the wave period of the instantaneous sediment transport induces a sediment transport rate 20% larger than using the same equation with the mean shear stress over the period. Moreover, as was observed previously, the phase-lag effects seem to be non-negligible, even for relatively small wave orbital velocities. This may explain the overestimation observed for the Ribberink formula.

4. Bed-load transport by waves and currents

The stirring effect due to waves in the presence of a steady current tends to increase the total sediment transport significantly. In order to generalize the proposed formula to encompass both waves and current, a new formula is proposed including the total shear stress obtained from the interaction between waves and current.

4.1. Development of a new formula

The conceptual model proposed for the case of waves only (cf. Eq. (15)) can be extended to the interaction between waves and current. Assuming that U_s is proportional to the shear velocity at the bottom, a $|\theta_{cw,onshore} - \theta_{cw,offshore}|^{1/2}$-dependence may be assumed for U_s, where the interaction between waves and current is taken into account. The representative shear stresses $\theta_{cw,onshore}$ and $\theta_{cw,offshore}$ are defined as the quadratic values of the instantaneous Shields parameter profile in the direction of the wave for the positive and the negative values of $\theta_{cw}(t)$, respectively (cf. Fig. 6b).

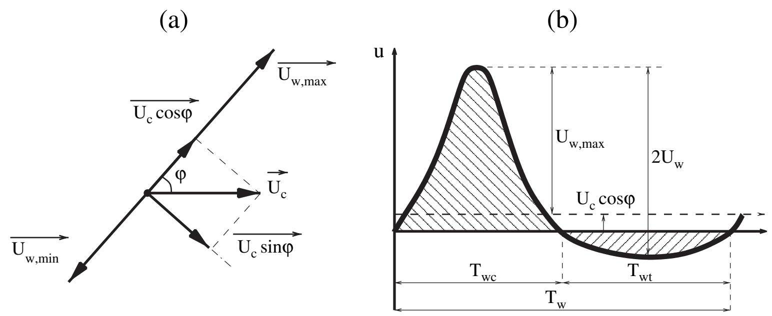

Fig. 6. (a) Definition sketch for wave and current interaction and (b) a typical velocity profile over a wave period in the direction of the waves including the effect of a steady current.

For an arbitrary angle φ between the waves and the current (Fig. 6a), this yields the same equations as in Eq. (13), where θ_w is replaced by θ_{cw}, and T_{wc} and T_{wt} are the half-periods where the instantaneous velocity $u(t) = U_c \cos\varphi + u_w(t)$ (or instantaneous Shields parameter) is onshore ($u(t) > 0$) or offshore ($u(t) < 0$), respectively (cf. Fig. 6). The instantaneous Shields parameter is defined as follows,

$$\theta_{cw}(t) = \frac{\frac{1}{2} f_{cw} |U_c \cos\varphi + u_w(t)| (U_c \cos\varphi + u_w(t))}{(s-1) g d_{50}} \tag{17}$$

where f_{cw} is the friction coefficient for an interaction between wave and current. This coefficient is assumed to be constant over the wave period as a first approximation, but as pointed out in Section 3.2, a temporal variation exists. Madsen and Grant (1976) suggested that f_{cw} could be obtained as a linear combination of f_c and f_w ($f_{cw} = X f_c + (1 - X) f_w$ with $X = |U_c|/(|U_c| + U_w)$).

Assuming that the moving sediment layer thickness Δ_s is proportional to the total mean bottom shear stress will give a $\theta_{cw,m}$-dependence for the transport rate (the stirring at the bottom by the waves and current together is mobilizing the layer). For an arbitrary angle φ between the waves and the current, the mean and maximum combined Shields parameters at the bottom $\theta_{cw,m}$ and θ_{cw}, respectively, are written,

$$\theta_{cw,m} = \left(\theta_c^2 + \theta_{w,m}^2 + 2\theta_{w,m}\theta_c \cos\varphi\right)^{1/2} \tag{18}$$

$$\theta_{cw} = \left(\theta_c^2 + \theta_w^2 + 2\theta_w\theta_c \cos\varphi\right)^{1/2} \tag{19}$$

where the mean (maximum) shear stresses from the waves and current were simply added together (vector addition). A more sophisticated approach would include the time variation of the waves in this addition.

In summary, a general equation for the sediment transport under waves and currents is,

$$\begin{cases} \Phi_w = a_w \sqrt{\theta_{cw,onshore} + \theta_{cw,offshore}}\,\theta_{cw,m} \exp\left(-b\dfrac{\theta_{cr}}{\theta_{cw}}\right) \\ \Phi_n = a_n \sqrt{\theta_{cn}}\,\theta_{cw,m} \exp\left(-b\dfrac{\theta_{cr}}{\theta_{cw}}\right) \end{cases} \tag{20}$$

where w and n correspond, respectively, to the wave direction and the direction normal to the wave direction, $\theta_{cn} = 1/2[f_c(U_c \sin\varphi)^2/((s-1)gd_{50})]$, and a_w, a_n and b are coefficients as before. The same value of $b = 4.5$ is kept. In order to be consistent with previous results obtained for current only and waves only, the following relationship is proposed for the coefficient a_w,

$$a_w = 6 + 6Y \quad \text{with } Y = \frac{\theta_c}{\theta_c + \theta_w} \tag{21}$$

and $a_n = 12$.

4.2. Comparison with experimental data

To investigate bed load transport where waves and current are interacting, several existing data sets were compiled and analyzed (see Table 3). Most of the data are from OWT experiments implying large orbital velocities with prevailing bed load transport, as previously discussed. More recently, Dohmen-Janssen and Hanes (2002) carried out some experiments in a large-scale wave flume (LWF). For non-breaking waves, they found that the bed load sediment transport represents around 90% of the total load. These experiments with real waves showed that the results are quite consistent with those observed in oscillating water tunnels (OWTs), although differences in the suspended sediment concentration and the total sediment transport rate are apparent. Net transport rates under waves were found to be about a factor 2.5 larger than in uniform horizontal oscillatory flows. They explained this by referring to the differences between boundary layer flows in OWTs and under free surface gravity waves. This difference may (partly) be attributed to the onshore directed boundary layer streaming that is present under waves and is absent in horizontal oscillatory flow.

In Fig. 7 is the bed load transport predicted by Eq. (20) plotted against experimental data for current and waves interacting (with $|U_c| > 0.02$). Good agreement is observed and more than 50% of the experimental cases are predicted within a factor of 2 of the measured values.

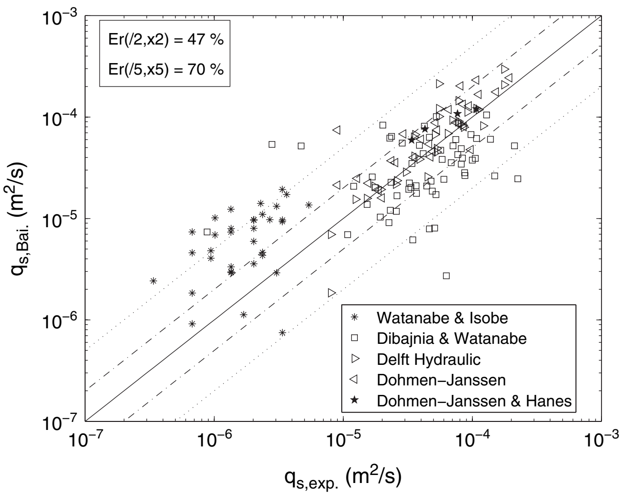

Fig. 7. Comparison between bed load transport predicted by the Eq. (20) and experimental data with current ($|U_c| > 0.02$ m/s) using the Wilson formula (1966 and 1989) to compute the roughness height ($k_s = k_{s,W}$).

A major part of the discrepancy ($E_{rms} \approx 2$, see Table 5) could be explained by the difficulties to predict roughness values for oscillatory sheet flows (no measured values available) and because phase-lag effects are not included in the formula (some large overestimations or even incorrect transport directions are observed when strong phase-lag occurs as in Fig. 5). The sediment transport rates for the Delft Hydraulic data set are slightly overestimated when the Wilson formula is used and better predicted using the skin roughness height. On the other hand, the sediment transport rates for the Dibajnia and Watanabe data set are underestimated when the skin friction is used and better predicted when the Wilson formula is used. The four data points from the large wave flume experiment are very well predicted (within a factor of 2 of the measured values). It seems that the differences observed by Dohmen-Janssen and Hanes (2002) between the LWF and OWT experiments for the linear relationship between the net transport rates and the third-power velocity moment are reduced by the integration of the friction coefficient (within the Shields parameter). Finally, contrary to the other formulas, good agreement is observed for the Watanabe and Isobe data set, corresponding to small wave orbital velocities and periods. This means that the effects of the critical Shields parameter are well described. It should be noted that for this data set, ripples were sometimes observed which induced stronger suspension and phase-lag effects. This may explain the opposite direction of the sediment transport observed for some of the data points as pointed out by Dibajnia and Watanabe (1992).

Table 5
Prediction of bed load transport rate within a factor of 2 and 5 of the measured values and root-mean-square errors using wave + current data ($|U_c| > 0.02$ m/s)

Author(s)	All data ($k_s = k_{s,W}$)			All data ($k_s = 2d_{50}$)		
	Px2 (%)	Px5 (%)	E_{rms}	Px2 (%)	Px5 (%)	E_{rms}
Bailard and Inman	47	70	2.07	41	68	2.10
Dibajnia and Watanabe	41	72	1.74	41	72	1.74
Ribberink	18	44	4.13	32	52	4.01
Eq. (20)	54	77	2.03	46	74	2.05

4.3. Comparison with existing formulas for waves and current

Only the Bailard and Inman, Dibajnia and Watanabe, and Ribberink formulas (cf. Section 3.1) include the effects of a current. In Table 5, the percentage of predicted values included within a factor of 2 or a factor of 5 deviation for the four studied formulas are presented for all the data where a mean current is present using the Wilson relationship ($k_s = k_{s,W}$) or the skin friction ($k_s = 2d_{50}$) to compute the roughness height.

It can be seen in Table 5 that a larger discrepancy exists compared to the steady current data as the total shear stress has to be estimated instead of using measured values but Eq. (20) still yields the best overall results. The Bailard and Inman formula overestimates the sediment transport rate for low shear stress (cf. the Watanabe and Isobe data set), and underestimates the rate for the Dibajnia and Watanabe data set and for all the cases where strong phase-lag occurs. Nevertheless, it presents quite good overall agreement with the data. As observed in Section 3.1, the Ribberink formula, as it stands, overestimates the transport rates when waves are present. Results are improved using the skin roughness height, but they remain poor compared to the other formulas. Finally, if the Dibajnia and Watanabe relationship shows good overall results, it overestimates the data from Delft Hydraulic and seems not to be sensitive enough to the bottom shear stress.

5. Conclusion

A new formula for bed load sediment transport was developed and presented that includes interaction between waves and current. This formula is based on the assumption that the sediment transport is proportional to the total Shields parameter to the power 1.5. For purely oscillatory flows, the mean Shields parameter for each half-period (when $u > 0$ and $u \leq 0$) is computed in order to take into account the effect of asymmetric waves. The new formula provides satisfactory agreement with the data sets that were compiled, and the best agreement compared to other formulas previously proposed.

The effect of the critical Shields parameter was studied more carefully and an exponential function of the ratio θ_{cr}/θ_{cw} was proposed. This relationship significantly improves the agreement for both steady current and wave cases.

The net sediment transport due to waves was smaller than expected. A coefficient value of $a = 6$ was found, although it reaches 12 for the steady current. This lower value may partly be explained by the fact that the mean shear stress over each half-period is used. Nevertheless, it seems that phase-lag effects are present even for small wave orbital velocities and coarser sediment, which introduce a lower net sediment transport over the wave period.

Furthermore, some discrepancy occurs since the total shear stress is unknown for many of the experiments. Indeed, for the experiments with oscillatory flows, the total shear stress has to be estimated based on theoretical values. Two calculation approaches were presented: using the Wilson formula to compute the roughness height, or using the skin roughness height (even if it is known that the roughness increases strongly when sheet flow occurs). Depending on the data set, the former or latter calculation yields the best agreements. It stresses how important it is to accurately estimate the bottom shear stress when sheet flow occurs in order to predict the sediment transport rate accurately.

Finally, as the formula does not take into account the effect of phase-lag, adding a coefficient quantifying this effect should increase the accuracy of the formula. The phase-lag phenomenon is the main non-steady effect due to oscillatory flows: a quantity of sand can still remain in suspension after each half-cycle of the wave velocity profile, and hence move in the other direction. Dibajnia and Watanabe (1992) introduced a semi-empirical formula which allows for the estimation of phase-lag effects. Dohmen-Janssen (1999) and Camenen and Larroudé (2003) also proposed some semi-empirical coefficients to estimate phase-lag effects.

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